EXTENSION OF THE QUASILINEAR METHOD OF DETERMINING THERMOPHYSICAL CHARACTERISTICS TO THE CASE OF ANISOTROPIC MATERIALS

Ya. A. Napar'in, N. A. Yarmonov, and N. A. Vdovin

UDC 536.22

The article reports on the initial part of the stage of regular thermal regime of a plate heated by a constant heat flux, and a method is suggested for determining thermal conductivity and thermal diffusivity.

The present article suggests applying the method to flat specimens; this will make it possible to retain the positive aspects of the method [1] and to broaden the class of investigated materials to include anisotropic ones of which the thermophysical characteristics (TPC) are determined.

We assume that in the general case the investigated material is anisotropic. Let us consider the equation of heat conduction

$$c\rho \frac{\partial T}{\partial t} = -\operatorname{div}\mathbf{q} + g. \tag{1}$$

For orthotropic bodies the value of q in a system of Cartesian coordinates is equal to:

$$\mathbf{q} = -\left(\lambda_x \frac{\partial T}{\partial x}\mathbf{i} + \lambda_y \frac{\partial T}{\partial y}\mathbf{j} + \lambda_z \frac{\partial T}{\partial z}\mathbf{k}\right), \tag{2}$$

where λ_x, λ_y, λ_z are the thermal conductivities along the axes of coordinates. On the assumption that λ_x, λ_y, λ_z does not depend on the coordinates of the points of the body, and with (2) taken into account, we write Eq. (1) in the form

$$c\rho \frac{\partial T}{\partial t} = \lambda_x \frac{\partial^2 T}{\partial x^2} + \lambda_y \frac{\partial^2 T}{\partial y^2} + \lambda_z \frac{\partial^2 T}{\partial z^2}. \tag{3}$$

We will solve Eq. (3) with the following assumptions:

1) we have a specimen in the form of a rectangular parallelepiped, and the origin of coordinates lies at the center of one of its sides so that

$$0 \leqslant x \leqslant l, \ -l_y \leqslant y \leqslant l_y, \ -l_z \leqslant z \leqslant l_z; \tag{4}$$

2) at the initial instant the temperature of the specimen is equal to the ambient temperature;

3) there are no internal heat sources (g = 0);

4) at the instant t = 0 on the entire side x = 0 a surface heat source with constant heat flux density q begins to act;

5) on the other sides heat exchange with the environment obeys Newton's law.

In view of the above-said the problem reduces to the solution of Eq. (3) with the boundary conditions

$$T|_{t=0} = T_0, \ \lambda_x \left.\frac{\partial T}{\partial x}\right|_{x=0} = -q, \ \left[\lambda_x \frac{\partial T}{\partial x} + \alpha_x (T - T_0)\right]_{x=l_x} = 0,$$

$$\left[\lambda_y \frac{\partial T}{\partial y} \pm \alpha_y (T - T_0)\right]_{y=\pm l_y} = 0, \ \left[\lambda_z \frac{\partial T}{\partial z} \pm \alpha_z (T - T_0)\right]_{z=\pm l_z} = 0. \tag{5}$$

Perm Polytechnic Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 55, No. 4, pp. 611-616, October, 1988. Original article submitted May 11, 1987.

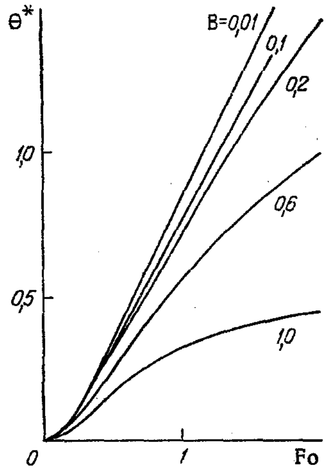

Fig. 1

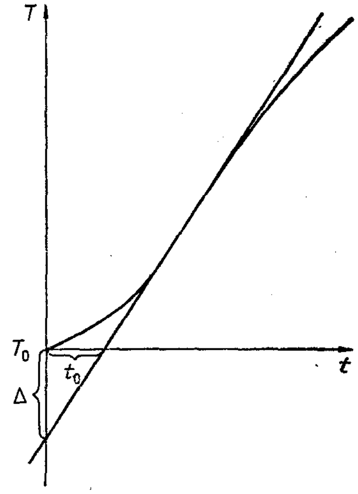

Fig. 2

Fig. 1. The dependence Θ*(Fo) on one surface of the plate when it is heated from the other side, with different conditions of heat exchange.

Fig. 2. On the processing of the experimental results.

The system (3)-(5) becomes greatly simplified if one of the dimensions of the specimen is made as small as convenient in comparison with the other dimensions. We introduce the dimensionless magnitudes:

$$\chi = \frac{x}{l_x},\ \eta = \frac{y}{l_y},\ \xi = \frac{z}{l_z},\ a_x = \frac{\lambda_x}{c\rho},\ \Theta = \frac{T - T_0}{T_0},$$
$$l_\eta = \frac{l_x}{l_y},\ l_\xi = \frac{l_x}{l_z},\ \lambda_\eta = \frac{\lambda_y}{\lambda_x},\ \lambda_\xi = \frac{\lambda_z}{\lambda_x},\ \mathrm{Fo} = \frac{a_x}{l_x^2} t \tag{6}$$

and rewrite Eq. (3) in dimensionless form

$$\frac{\partial\Theta}{\partial\,\mathrm{Fo}} = \frac{\partial^2\Theta}{\partial\chi^2} + \lambda_\eta l_\eta^2\ \frac{\partial^2\Theta}{\partial\eta^2} + \lambda_\xi l_\xi^2\ \frac{\partial^2\Theta}{\partial\xi^2}. \tag{7}$$

When we make ℓ_y and ℓ_z sufficiently large so that the coefficients ℓ_η and ℓ_ξ can be made as small as desired, we may neglect the last two terms in Eq. (7) and reduce the problem to the unidimensional case

$$\frac{\partial\Theta}{\partial\,\mathrm{Fo}} = \frac{\partial^2\Theta}{\partial\chi^2} \tag{8}$$

with the boundary conditions

$$\Theta|_{\mathrm{Fo}=0} = 0,\ \left.\frac{\partial\Theta}{\partial\chi}\right|_{\chi=0} = -Q,\ \left[\frac{\partial\Theta}{\partial\chi} + B\Theta\right]_{\chi=1} = 0, \tag{9}$$

where

$$Q = ql_x/T_0\lambda_x,\ B = \alpha_x l_x/\lambda_x. \tag{10}$$

By using the method of integral transformations, we obtain the solution of the system (8)-(9) in the form of the series

$$\Theta^* = 2\sum_{i=1}^{\infty} \frac{(\mu_i^2 + B^2)\cos(\mu_i\chi_i)}{\mu_i^2(\mu_i^2 + B^2 + B)}\,[1 - \exp(-\mu_i^2\,\mathrm{Fo})], \tag{11}$$

TABLE 1. Dependence of the Coefficients A and Θ_0 on the Conditions of Heat Exchange (numbers B)

B	0,01	0,05	0,10	0,20	0,30	0,40	0,50	1,00
A	0,9897	0,9578	0,9245	0,8687	0,8772	0,7816	0,7459	0,6120
$-\Theta_0$	0,1623	0,1517	0,1420	0,1277	0,1167	0,1079	0,1004	0,0754

where

$$\Theta^* = \lambda_x (T - T_0)/ql_x. \tag{12}$$

Summing is carried out over all the positive roots μ_i ($i = 1, 2, 3, \ldots$) of the characteristic equation

$$B \cos \mu - \mu \sin \mu = 0. \tag{13}$$

On the basis of practical considerations it is rational to measure the temperature on the side of the specimen $\chi = 1$. In that case, with a view to (13), we express the solution of (11) in the form

$$\Theta^* = 2 \sum_{i=1}^{\infty} (-1)^{i+1} \frac{\sqrt{\mu_i^2 + B^2}}{\mu_i (\mu_i^2 + B^2 + B)} [1 - \exp(-\mu_i^2 \, \mathrm{Fo})]. \tag{14}$$

An example of the dependence $\Theta^*(\mathrm{Fo})$ for $\chi = 1$ and different values of B is given in Fig. 1. We can see that on the graphs there is an inflection. In the vicinity of the point of inflection there is a segment of the dependence which in the limit coincides with the tangent at the point of inflection. We write the equation of the tangent

$$\Theta^* = \Theta_0 + A \, \mathrm{Fo}, \tag{15}$$

where A determines the slope tangent, and Θ_0 is the value of Θ^* for Fo = 0, i.e., the point of intersection of the tangent with the Θ^*-axis.

We determine the coefficient A from the condition

$$A = \left. \frac{\partial \Theta^*}{\partial \, \mathrm{Fo}} \right|_{\mathrm{Fo} = \mathrm{Fo}_n} = 2 \sum_{i=1}^{\infty} (-1)^{i+1} \frac{\sqrt{\mu_i^2 + B^2} \, \mu_i^2}{\mu_i (\mu_i^2 + B^2 + B)} \exp(-\mu_i^2 \, \mathrm{Fo}), \tag{16}$$

where Fo_n are the roots of the equation

$$\frac{\partial^2 \Theta^*}{\partial \, \mathrm{Fo}^2} = -2 \sum_{i=1}^{\infty} (-1)^{i+1} \frac{\sqrt{\mu_i^2 + B^2} \, \mu_i^4}{\mu_i (\mu_i^2 + B^2 + B)} \exp(-\mu_i^2 \, \mathrm{Fo}) = 0. \tag{17}$$

The coefficients Θ_0 are found from (15):

$$\Theta_0 = \Theta_n^* - A \, \mathrm{Fo}_n, \tag{18}$$

where

$$\Theta_n^* = \Theta^* |_{\mathrm{Fo} = \mathrm{Fo}_n}.$$

The coefficients A and Θ_0 for (16) and (18) are calculated with specified accuracy for different values of the number B. Table 1 presents examples of the values of A and Θ_0 for several values of the number B. Expression (15) can be used for determining the thermophysical characteristics. For that we express (15) in dimensional magnitudes

$$\frac{\Delta T}{ql_x} \lambda_x = \Theta_0 + A \frac{a_x}{l_x^2} t \tag{19}$$

and differentiate with respect to time

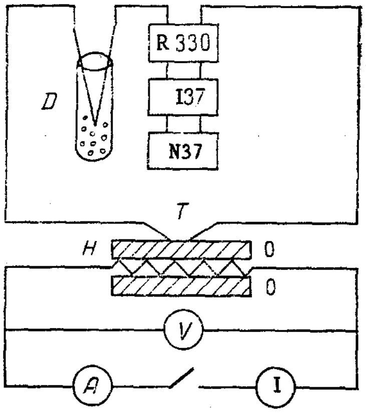

Fig. 3. Schematic diagram of the installation for determining the thermophysical characteristics of materials on specimens in the form of plates.

$$\frac{\lambda_x}{ql_x}\,\frac{d(\Delta T)}{dt}=\frac{A}{l_x^2}\,a_x. \tag{20}$$

Solving (19) and (20) for λ_x and a_x, we obtain the theoretical expressions:

$$a_x=\frac{\Theta_0 l_x^2}{A\left[t\,\dfrac{d(\Delta T)}{dt}-\Delta T\right]}\,\frac{d(\Delta T)}{dt},$$
$$\lambda_x=\frac{\Theta_0 q l_x}{t\,\dfrac{d(\Delta T)}{dt}-\Delta T}, \tag{21}$$

where $\Delta T = T - T_0$ is the difference between the temperature of the surface ($\chi = 1$) of the specimen and the ambient temperature. We express the theoretical expressions (21) through the characteristic magnitudes of the experimental dependence of the change of temperature on time. For that purpose we draw the tangent and extend it up to its intersection with the axes T and t (Fig. 2). The segments intercepted by the tangent on the axes are denoted Δ and t_0, respectively. It follows from geometric considerations that

$$\Delta T=t\,\frac{d(\Delta T)}{dt}-\Delta,\quad \frac{\Delta}{t_0}=\frac{d(\Delta T)}{dt} \tag{22}$$

and we write (21) in the form

$$\lambda_x=\frac{\Theta_0 q l_x}{\Delta},\quad a_x=\frac{\Theta_0 l_x^2}{At_0}. \tag{23}$$

The coefficients A and Θ_0 depend on the conditions of heat exchange (see Table 1), and the less heat exchange there is, the smaller is the error with which the limit values of the coefficients are taken: $A = 1$, $\Theta_0 = 1/6$.

For more accurate measurements of the thermophysical characteristics it is necessary to estimate the heat exchange realized in the experiment from the same thermogram (Fig. 2), and in accordance with the magnitude of the Biot number the values of the coefficients A and Θ_0 are taken.

A diagram of the installation for implementing the quasilinear method on flat specimens is shown in Fig. 3. The investigated specimens in the form of plates 40 × 40 × 5 mm in size were placed in a special measuring cell. For the investigation of the temperature dependence of the thermophysical characteristics the cell was put in a thermal chamber. Between the specimens the heater was inserted in the form of a flat spiral of constantan 0.05 mm thick. Current to the heater was supplied by a stabilized source VS-26, the power released by the heater was calculated from the voltage and current intensity measured with an ammeter M1107 and a voltmeter M1109. The instruments had class of accuracy 0.2. The surface temperature of the specimens was measured with the aid of a calibrated chrome-copel thermocouple with 0.09 mm diameter of the electrodes. One junction of the differential thermocouple was attached at the center of the surface of the specimen, the other was located in a Dewar vessel. The constant component of the emf arising in consequence of the temperature difference between the "hot" and the "cold" junctions was compensated by a potentiometer R330. The variable component arising after the heater had been switched on was transmitted to the dc amplifier I37 and was recorded on the recording strip of the instrument N37.

On the installation we measured the thermophysical characteristics of polymethyl metacrylate with a density of 1170 $kg \cdot m^{-3}$. Deviations of the TPC from the data of [2] did not exceed 6%; this confirms the efficiency of the method.

On the same installation we also measured the thermophysical characteristics of glass reinforced plastic in the direction perpendicular to the fibers. The filler in the investigated material was 50 weight parts glass tissue, the initiating system was benzene peroxide, binder was 40 weight parts polyester resin. The density of the investigated glass reinforced plastic was 1600 kg/m^3. Processing of the experimental data made it possible to represent the temperature dependence of the TPC of glass reinforced plastic for the temperatures 290-450°K in the form of the expressions:

$$a_T = (2{,}45 + 0{,}33 \cdot 10^{-2} T) \cdot 10^{-7}\ \mathrm{m^2/sec};$$

$$\lambda_T = (0{,}16 + 0{,}32 \cdot 10^{-3} T)\ \mathrm{W/m \cdot {}^\circ K};$$

$$c_T = (-0{,}48 + 0{,}52 \cdot 10^{-2} T)\ \mathrm{kJ/kg \cdot {}^\circ K}.$$

NOTATION

T, temperature of the plate; T_0, initial temperature of the plate and ambient temperature; g, volume density of the heat sources; q, heat flux density; ℓ_x, thickness of the plate; λ_x, thermal conductivity along the x axis; a_x, thermal diffusivity along the x axis; α, heat transfer coefficient; Fo, Fourier number; B, Biot number; ρ, density of the material of the plate; c, heat capacity; t, time.

LITERATURE CITED

1. Yu. A. Napar'in and N. A. Vdovin, Inzh.-Fiz. Zh., 29, No. 3, 436-440 (1975).
2. L. L. Vasil'ev and S. A. Tanaeva, The Thermophysical Properties of Porous Materials [in Russian], Minsk (1971).